MATH 54 - HINTS TO HOMEWORK 10

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Here are a couple of hints to Homework 10! Enjoy! :)

Note: If you're running out of time (or if you absolutely hate harmonic oscillators), you may skip the following problems:

(1) Section 4.7: You can skip the whole section!

(2) Section 4.9: Skip 1, 5, 7. Also, you don't have to sketch the solution in 3.

Note: This only applies to my discussion sections, this doesn't apply to the other sections! Also, the required problems in section 4.8 and 4.9 are important, so make sure to do them!

SECTION 4.7: QUALITATIVE CONSIDERATIONS FOR VARIABLE-COEFFICIENT AND NONLINEAR EQUATIONS

4.7.3. The book gives a physical explanation (which doesn't make sense to me), so let me give you a more mathematical explanation! Again, the word **qualitative** is important, which means your answer doesn't have to be rigorous at all!

Notice that $y'' = 6y^2 \geq 0$. Hence the function y is concave up everywhere!

This says that y should look like the parabola $y = x^2$. Moreover, since $y(0) = -1$ and $y'(0) = -1$, we know that y starts at -1 and then starts to decrease. However, because y is concave up and y is decreasing so quiclly, at some point y has to attain a minimum, and then increase without bounds! (because as y is large, y'' is large, so y increases faster and faster)

Note that this agrees with the corresponding graph!

4.7.7. Ughhhh, sorry, but this is waaay too much physics! :(

4.7.12. This is just a matter of plugging in y_2 into Legendre's equation

4.7.13. Just compare the graphs with figures $4.13, 4.16, 4.17$

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SECTION 4.8: A CLOSER LOOK AT FREE MECHANICAL VIBRATIONS

4.8.1, 4.8.9, 4.8.11. Use the equation $my'' + by' + ky = 0$, where:

(1) m is mass
(2) b is damping
(3) k is stiffness

Also, beware that the problems give you initial conditions!

4.8.1. $m = 3$, $k = 48$, $b = 0$. $y(0) = \frac{1}{2}$, $y'(0) = 2$.

The period is $\frac{2\pi}{\omega}$ and the frequency is $\frac{\omega}{2\pi}$, where ω is the term you find in the $\cos$ and $\sin$ terms in your solution (for example, if your solution involves $\cos(2t)$, then $\omega = 2$).

The amplitude is $\sqrt{C_1^2 + C_2^2}$, where C_1 and C_2 are the two constants in your solution.

Finally, you need to find the first time t such that $y(t) = 0$.

4.8.3. Notice that your solution is different depending on whether $0 \leq b < 8$, $b = 8$, or $b > 8$.

4.8.9. $m = 2$, $k = 40$, $b = 8\sqrt{5}$, $y(0) = 10$, $y'(0) = 2$. First find your solution, and then solve $y'(t) = 0$ (this might involve tan^{-1}, see example 3).

4.8.11. $m = 1$, $k = 100$, $b = 0.2$, $y(0) = 0$, $y'(0) = 1$. First find your solution, and then solve $y'(t) = 0$ (this might involve tan^{-1}, see example 3).

SECTION 4.9: A CLOSER LOOK AT FORCED MECHANICAL VIBRATIONS

4.9.1. Use your calculator!

4.9.3. Careful! Here one of the roots of the auxiliary equation, $r = 3i$ coincides with the term on the right-hand-side, $2\cos(3t)$, hence you have to guess $y_p(t) = At\cos(3t) + Bt\sin(3t)$ (hence the term 'resonance')

4.9.5. First divide the equation by m, then find the homogeneous solution, and then find a particular solution of the form $y_p(t) = A\cos(\gamma t) + B\sin(\gamma t)$ (no t factors because $\gamma \neq \omega$).

The 'trigonometric identity' the book talks about is:

$$\cos(A)\cos(B) = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

Finally, (c) looks long, but all you have to do is to sketch the curve in (b). Use your calculator!

4.9.7. This looks bad, but it's not as horrible as you might think! First of all, $y(t) = y_0(t) + y_p(t)$ as usual, but here $y_0(t)$ is given by formulas (18) and (19) on page 509. Moreover, y_p stays **exactly** the same!

SECTION 6.1: BASIC THEORY OF LINEAR DIFFERENTIAL EQUATIONS

6.1.1, 6.1.3. First of all, make sure that the coefficient of y''' is equal to 1. Then look at the domain of each term, including the inhomogeneous term (more precisely, the part of the domain which contains the initial condition -2 resp. 5). Then the answer is just the intersection of the domains you found!

6.1.9. $\cos^2(x) + \sin^2(x) = 1$, so linearly dependent

6.1.11. Use the Wronskian with $x = 1$

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6.1.23. For example, for (a), we have:

$$L\left[2y_1 - y_2\right] = 2L\left[y_1\right] - L\left[y_2\right] = 2x\sin(x) - (x^2+1) = 2x\sin(x) - x^2 - 1$$

So $2y_1 - y_2$ solves the equation for (a)

6.1.27. Either you can use the Wronskian with $x = 0$ (the matrix becomes a diagonal matrix with (n,n)th term n), or use the following reasoning: If

$$a_0 + a_1 x + a_2 x^2 \cdots + a_n x^n = 0$$

This means that for **EVERY** x, x is a zero of $a_0 + a_1 x + a_2 x^2 \cdots + a_n x^n$ (by definition of the zero function). However, this polynomial is of degree n, hence cannot have more than n zeros unless $a_1 = a_2 = \cdots = a_n = 0$, which we want!

SECTION 6.2: HOMOGENEOUS LINEAR EQUATIONS WITH CONSTANT COEFFICIENTS

6.2.1, 6.2.3, 6.2.5, 6.2.7, 6.2.9, 6.2.11, 6.2.13. The following fact might be useful:

Rational roots theorem: If a polynomial p has a zero of the form $r = \frac{a}{b}$, then a divides the constant term of p and b divides the leading coefficient of p.

This helps you 'guess' a zero of p. Then use long division to factor out p.

6.2.15, 6.2.17. The reason this is written out in such a weird way is because the auxiliary polynomial is easy to figure out! For example, in 6.2.15, the auxiliary polynomial is

$(r-1)^2(r+3)(r^2+2r+5)^2$.

6.2.25. Suppose:

$$a_0e^{rx} + a_1xe^{rx} + \cdots + a_{m-1}x^{m-1}e^{rx} = 0$$

Now cancel out the e^{rx}, and you get:

$$a_0 + a_1x + \cdots + a_{m-1}x^{m-1} = 0$$

But $1, x, x^2 \cdots, x^{m-1}$ are linearly independent, so $a_0 = a_1 = \cdots a_{m-1} = 0$, which is what we wanted!